

COMMUNICATIONS IN CURRICULUM MATERIALS AND TYPES OF STUDENT AUTONOMY PROMOTED

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The interaction between types of communication and socio-mathematical norms on students' learning autonomy in five curriculum programs was investigated. Uni-directional, contributive, reflective, and instructive types of communications were present in curriculum programs studied. Investigations in Number Data and Space and Math Trailblazers provided opportunities for students to explain, justify, and compare solution strategies. Math in Focus and Scott Foresman Addison Wesley-Mathematics required explanations, justifications, and comparisons of solution strategies, but those were mainly provided by the teacher. The former and latter programs potentially foster intellectual autonomous and intellectual heteronomous learning in students, respectively, while Everyday Mathematics almost equally supports both.

Key words: Curriculum Analysis; Classroom Discourse; Curriculum

Classroom discourse is critical for sharing, reflecting upon, refining, supporting, and extending students' mathematical ideas (NCTM, 2014), and the ways in which teachers organize classroom discussion influence student thinking. Curriculum materials (CMs) communicate to teachers in ways that can position students as largely independent learners, as learners depending heavily on an authority, or as both types of learners during enactment. Yackel and Cobb (1996) described students who “make sense of explanations, compare solutions, and make judgments about similarities and differences” (p. 466) as intellectually autonomous and otherwise as intellectually heteronomous (i.e., relying heavily on an authority to lead and determine mathematically appropriate ways to act). These descriptions by Yackel and Cobb imply that kinds of engagement opportunities offered to students in the classroom determine the degree of independence provided to students in the learning process. Yackel and Cobb also recommended teachers establish what they call socio-mathematical norms—“normative aspects of mathematical discussions that are specific to students' mathematical activity” (p. 458) in their classroom so that students can decide for themselves what is mathematically appropriate and acceptable with minimal mathematical input from the teacher. Little is known about the interaction between communication types and socio-mathematical norms embedded in CMs and their impacts on students' autonomy. This study examines CMs for types of communication they promote and the kind of learning autonomy fostered. To investigate this, we asked a research question: *What are potential impacts of communication types and socio-mathematical norms embedded in written lessons?* Three reasons suggest that investigating the potential impacts of the interaction between communication types and socio-mathematical norms might benefit the mathematics education community.

First, students need to develop authority in the mathematics they learn, and teachers are in the position to foster desired “students' mathematical authority” (Stein, Engle, Smith, & Hughes, 2008, p. 332). Teachers might be able to do this when curriculum designers embed such practices in CMs. Examining the impact of communication types embedded in written lessons might explain the kind of mathematical authority teachers are expected to promote in students.

Second, Hiebert, Morris, Berk, and Jansen (2007) argued that teachers ought to learn from teaching in order to improve their practice. A possible way of doing this is by communicating to teachers through CMs what counts as a mathematical explanation and justification, which has possibility to develop “teachers' mathematical knowledge for teaching” (Ball, Thames, & Phelps,

2008, p. 394). As teachers understand what counts as mathematical explanation and justification, they might be able to orchestrate classroom interaction to develop mathematical authorities students need to be successful.

Third, Shulman (1986) recommended that teachers should understand both substantive and syntactic structures of mathematical ideas students are to learn. An understanding of ways CMs communicate, what is communicated, and the potential impact on students' mathematical authority may provide insights into whether teachers' learning of substantive and syntactic structures of mathematics is promoted.

Theoretical Perspectives

Researchers have investigated classroom discourse to identify ways in which teachers communicate and how these might affect students' mathematical thinking. Brendefur and Frykholm (2000) identified four hierarchical types of communication between two preservice teachers and their students: *uni-directional*, *contributive*, *reflective*, and *instructive*. They defined uni-directional communication as dominated by teachers; contributive as limited to sharing of ideas with minimal attention given to discussing them; reflective communication makes shared ideas objects of discussion; and instructive communication focuses on students' thought processes that reveal their strengths and limitations. Brendefur and Frykholm argued that classifying these four types is important because they affect classroom norms.

Cobb, Yackel and Wood (1989) and Yackel, Cobb, and Wood (1991) identified examples of socio-mathematical norms in classrooms as explanations, justifications, and argumentation. Yackel and Cobb (1996) argued that socio-mathematical norms focusing on "what counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant ... acceptable mathematical explanation and justification" (p. 461) create additional learning opportunities for students and can develop their mathematical thinking.

CMs can contribute to teachers' classroom practice by promoting productive communications among teachers and students (Ball & Cohen, 1996; Davis & Krajcik, 2005) and supporting them to establish appropriate socio-mathematical norms in the classroom. As such, it is natural to ask whether CMs explicitly support teachers for effective classroom discourse, in terms of socio-mathematical norms and different types of communication. Because of the critical role CMs can play in influencing classroom practice, we drew on the works of Brendefur and Frykholm (2000) and Yackel and Cobb (1996) to identify types of communications embedded in CMs and potential impacts of communications types and socio-mathematical norms.

Methods

Data for this study were drawn from five curriculum programs used in a larger project, *Improving Curriculum Use for Better Teaching (ICUBiT)*. We selected written lessons from each of the five curriculum programs: (a) *Investigations in Number, Data, and Space (Investigations)*; (b) *Everyday Mathematics (EM)*; (c) *Math Trailblazers (MTB)*; (d) *Scott Foresman Addison Wesley-Mathematics (SFAW-Mathematics)*, and (e) *Math in Focus (MiF)*. The first three programs are reform-oriented and their designs were funded by the National Science Foundation; the fourth is commercially developed and widely used in U.S. classrooms; and the fifth, originally from Singapore, is gradually and steadily gaining prominence in U.S. classrooms. Teacher's guides of these five curriculum programs were analyzed, focusing on the guidance provided in the main portion of lessons—devoted to the main content, excluding routine practice. Fifteen written lessons in grades 3-5 (five per grade) randomly selected from each program were analyzed.

The main parts of each lesson were coded sentence by sentence and the analysis went through five stages. First, each sentence was associated with one of the following codes: (1) Uni-directional – directly speaking to teachers or students (through teacher); (2) contributing –explaining and

demonstrating possible ideas; (3) reflecting – engaging students in making sense and generating meaning through use of representations, strategies, and discussions; and (4) instructing –posing situations that encourage students to compare and make judgments. Second, each sentence was again coded for socio-mathematical norms (i.e., forms of students' engagements): (1) making sense of explanations; (2) comparing solutions and strategies; (3) making judgments about similarities and differences; and (4) providing explanations, demonstrations, and justifications. Third, we made a summary for the number and percentage of sentences for each type of communication. Fourth, within each type of communication, we used socio-mathematical norms promoted to determine the degree of possible students' autonomy fostered in each program. When CMs communicate to teachers to engage students in making comparisons of solutions and strategies, providing explanations, demonstrations, or justifications, students appear to be supported to be more intellectually autonomous. Otherwise, they may be moving toward the intellectually heteronomous end, depending on the teacher or an authority to understand and articulate mathematical concepts or ideas. Fifth, for each program, we identified patterns in types of communication promoted and described the kinds of autonomy fostered, as described above, and compared the five programs in terms of kinds of communications and the type of student learning autonomy the programs tend to support.

Results

Table 1 shows that all four types of communications are present in all five curriculum programs, but with varying emphasis. For each program, the most dominant type of communication is *uni-directional*. *Contributive* communication is the least emphasized

Table 1: Percentages of Communication Promoted

Types of Communication	<i>INVESTIGATIONS</i>	<i>SFAW-MATHEMATICS</i>	<i>EM</i>	<i>MTB</i>	<i>MiF</i>
Uni-directional	60.0	80.5	76.9	84.1	70.2
Contributive	9.0	3.5	4.7	4.6	10.1
Reflective	16.1	5.6	10.7	6.2	6.4
Instructive	14.9	10.4	7.7	5.1	13.3
Grand Total	100	100	100	100	100

type in all of these programs, although *MiF* allocated a greater portion of its guidance to this type of communication, followed by *Investigations*. However, most of the contributions in *MiF* are from the teacher, whereas in *Investigations* they are from students. The proportion of sentences allocated for *reflective* communication is greatest for *Investigations* and least for *SFAW-Mathematics*. *Investigations* provide students opportunities to provide explanations, justifications, comparison of solution methods, and strategies presented, whereas in *SFAW-Mathematics* these are often provided by teachers. The proportion of sentences allocated for *instructive* communications is greatest for *Investigations* and least for *MTB*. In spite of this proportional difference, *Investigations* and *MTB* lessons guide teachers to use students' thinking to shape subsequent lessons. In other words, what students struggle with is used to design subsequent lessons.

The emphasis on types of communication propagated by each curriculum program indicates the kind of autonomy nurtured in student learning. Figure 1 shows a continuum from intellectually heteronomous to intellectually autonomous and the location of each curriculum program used for this study along this continuum.

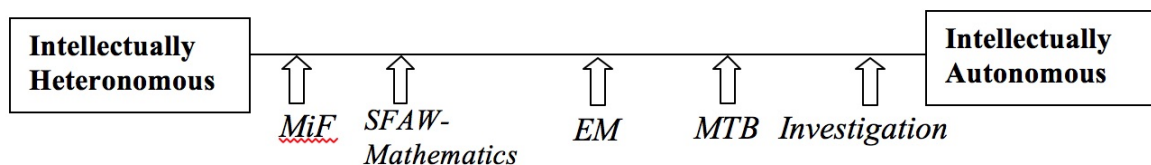


Figure 1: Intellectually Heteronomous-Intellectually Autonomous Continuum.

Intellectually Autonomous

All types of communications in *Investigations* provide students with opportunities to *justify*, *demonstrate*, and *make sense*. Contributive communications in *Investigations* position students to provide their solution strategies and methods, *justifying* and *demonstrating* to others. For example, *Investigations* provides an anticipated student's explanation of how a 120-degree angle is formed as "I put four 30-degree angles (on shape O) together to make 120, because 4 times 30 is 120," which also includes a *justification* of why the strategy and reasoning is mathematically appropriate. Students are also expected to demonstrate their solutions in class with others, as *Investigations* directs teachers to "ask students to demonstrate each of their strategies with the Power Polygon pieces they used." As students provide explanations, justifications, and demonstrations, other students are provided with opportunities to make sense, compare, and identify similarities and differences with theirs.

Instructive communication in *Investigations* provides questions to teachers to determine whether students are *making sense* and *justifying* their solutions. For example, "Do students correctly identify each angle? Do they use what they know about the measure of other angles, such as right angles, to help them find the measure of new angles?" These questions when posed to students provide opportunities to compare and make judgments. Therefore, *Investigations* seems to position students to rely more on their capability, thereby making them intellectually autonomous.

In *MiF*, instructive communication emphasizes *comparing solution methods* to determine which is simpler. During Guided Practice, *MiF* directs teachers to "Have students work in pairs to solve the problem. Have each pair choose a method to solve it. Then have students explain why their method is simpler." In this task, students are required to compare solution methods presented to them by the teacher and determine which is more efficient.

In *EM*, reflective communication suggests that students compare their work, such as "before turning in their work, have students compare their answers with a partner." This has the potential of moving students beyond just comparison of answers to reflecting on their solution strategies in case their answers are different or they have different solution methods to understand mathematical ideas in them. In *MTB*, reflective communication demands that students support their thinking. Questions are provided that asks for mathematical justifications from students for what decision they make. For example, questions such as, "Which fraction is larger: $\frac{1}{6}$ or $\frac{1}{4}$? How do you know? Show me with circles," are suggested to teachers to pose to students. *MTB* further asks teachers to "tell students that they are going to defend their choice to the class." These questions could potentially modify students' mathematical understanding by causing them to reflect on a representation and provide substantive arguments to support their reasoning. *MTB* directs teachers to position students to be reflective by letting them know that whatever choice they make needs to be defended. This, in a way, increases the "cognitive demand" (Stein, Grover, & Henningsen, 1996) of the tasks students are engaged with.

MTB supports the use of multiple strategies in classrooms but these come from students, in contrast to *MiF*. For example, *MTB* provides sentences such as, "Did anyone think of it in another way? Can you use another tool?" These different strategies contributed by students could enable them to build their confidence in doing mathematics. It also provides multiple entry points by which

students come to understand a mathematical idea rather than depending on the unique approach usually provided by teachers. It also communicates to students that in the absence of the teacher's direct guidance, they could devise valuable and appropriate solution strategies and move their own learning forward.

In some cases, the authors of *SFAW-Mathematics* recommend that teachers provide reflective opportunities to students by asking questions such as, "Suppose the survey question in Example A had been asked to 50 people at a dog show. Would the sample represent the entire population well? Explain." Such questions position students to think about what a sample of a population means and whether the selected sample can adequately represent a population. It also positions students with opportunities to reflect on how a sample can be selected so that it adequately represents that population from which it is drawn. An important idea students must understand before making such a reflection is, what is a population? The authors of *SFAW-Mathematics* direct teachers to explain what an entire population is, in case students do not understand it (example provided under intellectual heteronomous). Although reflective opportunities are provided to students, the authors of *SFAW-Mathematics* often take away such moments and immediately hand them over to the teacher.

Intellectually Heteronomous

In *MiF*, contributive communication dominates, but solution strategies and methods presented are mainly from the teacher. For example, sentences such as "Show students how to rename a mixed number as an improper fraction using multiplication and addition with $3\frac{1}{2}$ as an example" are used often, and this is followed by the methods which must be demonstrated by the teacher. In *MiF*, reflective communication emphasizes *comparison* of methods used, but the comparison is done mainly by teachers. For example, teachers are asked to "compare this method to the method in the previous *Learn*. Lead students to see that both methods involve multiplication followed by addition." The teacher is expected to *make judgments about similarities and differences*, and learning opportunities for students to develop these understandings themselves are lost. This implies that in *MiF*, students are not provided with opportunities to make sense of solution strategies presented and provide justifications why the methods work. Therefore, students seem to be frequently positioned in a way that makes them rely heavily on an authority, mainly the teacher.

In some situations, *EM* solely positions teachers to be the main authority in class, providing vital mathematical definitions and explanations, and demonstrating solution methods to students. For example, *EM* directs teachers to "explain that a map scale is a tool that helps to estimate real distances between two places shown on a map by relating the distances on the map to distances in the real world." Also, *EM* asks teachers to "model the following solution methods in your discussion." At other times, *EM* provides a step-by-step approach for teachers to lead students through to their solution. The methods teachers are asked to model above for students illustrate this approach. Although these are instructive, most of it comes from the teacher, communicating to teachers that students must depend on them.

In *SFAW-Mathematics*, instructive communication is mainly for teachers to direct students on what to do or provide explanations of mathematical concepts for student learning. The authors of *SFAW-Mathematics* speak to teachers about possible modifications that should be made to foster students' learning of mathematics. For example, they communicate to teachers that "if students do not understand what is meant by the entire population, explain that this is the whole group of people being considered by those who are conducting the survey. For example, it might be everyone living in the United States over the age of 18." Also, the authors of *SFAW-Mathematics* identify possible errors students might make and suggest ways teachers might engage students in fixing them, instead of recommending an instructive approach that positions students to depend on the teacher's authority. For example, *SFAW-Mathematics* authors suggest that "if students cannot decide whether

statements are fact or opinion, then point out that words such as best, good, and favorite are clues that a statement is probably an opinion.” In this way, the teacher is positioned as the main classroom authority students must look up to in times of difficulties or challenges on certain mathematical ideas. As such, these students might be given the signal that the teacher must evaluate every response as correct or incorrect before they can proceed. Students exposed to this kind of approach might not develop the ability to determine for themselves whether an approach or argument is mathematically sound and accurate.

Discussion/Significance

Many researchers have described curriculum materials as carriers of educational reform because whatever ideas for improvement are conceptualized, they must pass through these materials to get into classrooms. One of the goals of reform efforts has been to make students independent learners. In other words, reform efforts seek to develop students’ understanding of mathematical concepts as they reason and provide justification for why an explanation is accurate, decide for themselves what is mathematically correct and acceptable, and critique the reasoning of others students (NCTM, 2000) to learn independently and be certain progress is made. In order for this to happen, teachers need to create such a learning environment. NCTM (2000) recommended that teachers should create environments “in which intellectual risks and sense making are expected” (p. 197).

In this study, some curriculum materials have been found to communicate to teachers in ways that make intellectual risk taking a substantial part of classroom discourse, while others have minimally done so. For example, the authors of *Investigations* and *MTB* have provided teachers with ways students might be thinking about particular mathematical concepts and the depth of the ideas embedded in students’ thoughts. The authors of these two curriculum programs have also provided questions teachers might ask to help engage or motivate students to take intellectual risks. In other words, these two programs have communicated the risk-taking environments by providing what students may say and how ideas presented can be deliberated upon. These programs have opened up opportunities for students to justify whatever they say and embrace critique from their peers. These two programs have the potential to empower teachers to support students in figuring out their difficulties and making judgments while depending less on the teacher. Hence, when teachers implement suggestions from curriculum programs similar to these two, focusing on learning goals, students in such classrooms are likely to develop intellectually autonomous habits of mind, monitoring their own progress and making substantial claims of what they understand and what challenges them.

Other curriculum programs such as *MiF* and *SFAW-Mathematics* have attempted to provide opportunities for explanations and justification, but mainly by the teacher. As mentioned above, these programs often require teachers to explain mathematical ideas and strategies to students as well as provide justifications for why approaches they demonstrate are mathematically accurate. These programs implicitly communicate to teachers to take absolute control of the mathematics taking place in their classes. As such, students are to follow, depending heavily on the teacher to “show” them what is correct to copy. Such students are likely to depend greatly on the teacher or any other authority, possibly making them intellectually heteronomous. This does not mean students taught this way might never become intellectually autonomous, but that it might take them a much longer time, thereby achieving this retrospectively and delaying learning of other mathematical concepts. Cumulatively, delays of this sort might have a long-term negative impact on students’ learning, causing students to be discouraged about mathematics.

Although many studies have emphasized the kinds of resources CMs make available to teachers (e.g., Ball & Cohen, 1996; Davis & Krajcik, 2005), the study reported here extends our understanding of the kinds of resources that CMs offer to teachers. In addition to overall organization, development of content, and how this content should be taught (e.g., Davis & Krajcik,

2005), mathematics CMs also offer ways teachers might communicate to students to support their learning of mathematical content. Brendefur and Frykholm (2000) identified four types of communication teachers use in their classroom to interact with students. The study reported in this paper found that these four types of communications are provided in all CMs mentioned above. Although Brendefur and Frykholm did not indicate the kind of curriculum materials used by teachers in their study, it is possible the teachers may have been drawing on resources/guidance from their CMs to communicate with their students. Teachers are therefore encouraged to pay close attention to ways CMs communicate with them and how CMs expect them to communicate with students in order to promote student learning, as the communication types identified by Brendefur and Frykholm have been found to create learning opportunities for students if judiciously used.

This study found that CMs can foster both kinds of learning autonomy, although some may strongly propagate one type or the other or even both in some significant proportion (like *EM*). Reform efforts (e.g., NCTM, 2014) support intellectual autonomy for students by recommending that effective teaching provide opportunities for students to share ideas, clarify mathematical understanding, justify their approaches, and also critique ideas of others. This recommendation underscores the importance of autonomous learning that students might achieve and teachers need to support to effectively make student learning of mathematics autonomous. One way to support teachers to gain such knowledge to promote students' autonomous learning of mathematics is by embedding such moves in the teachers' guide. Although some curriculum programs have begun doing this, as the results of this study indicate, it is important to note that curriculum designers may make use of this finding to develop better ways of promoting students' learning autonomy and communicate these to teachers effectively. Hence, these findings are potentially useful for curriculum designers to deliberately position students on the path of intellectual autonomy.

These findings are also potentially useful for teacher educators. Although during teacher training preservice teachers often take a look at curriculum programs, they have done so mainly in the light of preparing lessons to teach during field work. Rarely have preservice teachers been focused on examining the curriculum in the direction of promoting students' learning autonomy. Teacher educators might use these results to engage their preservice teachers in examining curriculum programs for how teachers are positioned to promote learning autonomy in students beyond simply asking students to engage with tasks. This can potentially support teachers to gain skills in assessing curriculum materials for use in schools based on which kind of learning autonomy is fostered.

Providing students opportunities to share their solutions, explain their work, make comparisons, and justify their thinking might not automatically develop the needed intellectual autonomy for students. Although this study has revealed the relationship between types of communications and desired intellectual autonomy and identified the pathway to sustained intellectual autonomy, the following questions still need to be answered: How can teachers effectively foster students' intellectual autonomy in the classroom environment of sharing, comparing, explaining, and justifying reasoning? This question is important because teachers must be able to assess and conclude that their students can work independently and still make sense. Further research involving many classroom observations over an extended period of time might provide results that can fully describe and characterize intellectual autonomy.

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